Abstract

The project consists in designing a spiking neural network supporting a Theta spontaneous rhythm activity with nested Gamma oscillations, like the one observed in CA1 in the hippocampus. Inhibitory mean-field models or mechanisms at the level of intrinsic properties in the neuron already exist to explain this kind of bimodal activity, but we propose here to derive the predictive coding neural equations in order to find sets of heterogeneous lateral connectivities in Leaky-Integrate-And-Fire neurons which also contribute to do so. Further explorations of the model would consist in determining if the predictive framework is also naturally designed to reproduce more details in the CA1 power spectrum features (amplitude of the Gamma oscillations during one Theta cycle, distribution of the firing rate phases in the population...)

Model

Classic predictive equations

We will use a spiking model network of N LIF neurons whose equations are obtained from the predictive spiking framework. In this context, the network tries to reconstruct a M-dimensional sensory $x(t)$ signal which is itself obtained from a linear differential system:

$$\dot{x}(t) = Ax(t) + c(t)$$  \hspace{1cm} (1)

where $c(t)$ are the commands of the system, and $A$ the jacobian matrix that transforms the input into a specified dynamic that is our target one. At the same time, the neurons receive the same commands in their input plus a certain lateral connectivity (to be determined), and we want the sensory signal reconstructable from the output firing rates $o(t)$, with a linear decoder matrix $\Gamma$ and estimator variables $\hat{x}(t)$ so as:

$$\dot{\hat{x}}(t) = -\lambda_D \hat{x}(t) + \Gamma o(t)$$  \hspace{1cm} (2)

where $\lambda_D$ is the frequency scale of the decoder. The connectivity between the LIF neurons is derived from the minimisation of this cost function:

$$L(t) = \frac{1}{2} ||x(t) - \hat{x}(t)||^2$$  \hspace{1cm} (3)

Derivation of Eq.(3) like a gradient descent lead to write the voltages $V(t)$ of the neurons so as:

$$\dot{V}(t) = -\left(\frac{1}{\tau_m}\right)V(t) + \Gamma^T(A + \lambda_D I)\Gamma(o(u) * e^{-\lambda_D u})(t) - \Gamma^T \Gamma o(t) + \Gamma^T c(t)$$  \hspace{1cm} (4)

with a threshold for each neuron $i T_i = \frac{1}{2} \Gamma_i^2$. In this model, note that the neurons are connected with a fast current $\Gamma^T \Gamma$ $a,d$ a slow one $\Gamma^T(A + \lambda_D I)\Gamma(o(u) * e^{-\lambda_D u})(t)$. 
Lateral connectivity helping for a bimodal oscillating activity

First part of the project would consist in designing basic $A$ jacobian matrix that helps the dynamic of $\hat{x}$ to have resonance in the Gamma and Theta frequency ranges. Notably any kind of $A$ matrix builded with blocks and synaptic loops with negative feedbacks is a potential instance, for example the matrix $A$ with a disynaptic EI loop and an inhibitory trisynaptic one is in our interest.

$$A_{12,345} = \begin{pmatrix}
0 & -A_{12} & 0 & 0 & 0 \\
A_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -A_{345} & 0 \\
0 & 0 & 0 & 0 & -A_{345} \\
0 & 0 & -A_{345} & 0 & 0
\end{pmatrix}$$  \hspace{1cm} (5)

This shape of $A$ has a significant effect on the structure on the slow lateral $\Gamma^T(A + \lambda I)\Gamma$ connectivity, and confers to the sensory variables resonant properties at the inverse of the time-scale for the current to be propagated in the looped paths. As the neuronal networks is builded to make the estimators variables tracking the sensory variables, one can expect that the bimodal resonant properties will be also seen at the scale of the firing rates and the outputs of the LIF networks.